DETERMINATION OF INFLUENCE LINES AND ELASTIC PROPERTIES

by

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SYNOPSIS

Influence lines for statically indeterminate structures may be obtained by several different numerical procedures, all of which are based on the principles of elementary structural mechanics. Using some of the same basic principles in a more efficient and organized procedure can reduce a somewhat complex structural problem to one which is accurately and efficiently solved.

The procedure set forth in this report is based on the Müller-Breslau Principle[2] combined with the Moment Distribution Method and is very adaptable to beams which are continuous over more than two simple supports. Although the procedure is best suited for members with non-uniform cross-section, it is also valid for members with uniform cross-section.

When the sectional variation becomes difficult to express as a function of x, as is the case in many indeterminate beams with a variable moment of inertia, the use of elastic weights applied to the conjugate beam lends itself readily.

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1 Graduate Student, Department of Civil Engineering, Kansas State University, Manhattan, Kansas.

2 "Elementary Structural Analysis," by Charles Norris and John Wilbur, McGraw-Hill Book Co., New York, N.Y., 2d Ed., 1960, p. 493.

The modified stiffness, true stiffness, and the carry-over factors are determined for each end of each span. With these values, a unit moment applied at each end of each span can then be distributed by the Moment Distribution Method. Using the resulting moments to combine the fixed-end moment influence lines, the influence lines for the moment at any interior support may be determined. With such influence lines determined it is relatively easy to calculate any other influence lines, for moment, shear, or reactions, by using the equations of statics.

In order to illustrate the method used in determining the influence lines for the support moments and the elastic properties an example problem is worked in detail. In the example problem, for simplicity, the span lengths are equal and each span is symmetrical about mid-span.

INTRODUCTION

The basic concept of an influence line for an indeterminate structure is the same as that for the determinate structure. In either case, the influence line may be defined as a diagram whose ordinates give the value for a stress function (shear, moment, or reaction) when a unit load is placed on the structure at the same point as the ordinate.

Although the ordinates of an influence line for any stress element may be obtained by placing a unit load successively at each load point on the structure, this procedure becomes a long

and tedious process. Also, Müller-Breslau[3] procedures for determining influence lines may often be laborious, especially if an indeterminate structure remains after the initial redundant has been removed.

The method described herein is a combination of basic principles and procedures, so organized as to solve the problem accurately and efficiently. Although this method is adaptable to any indeterminate structure, a continuous beam with a variable moment of inertia is used to outline the general procedure.

GENERAL PROCEDURE

Each span of the continuous beam shown in Fig. 1a is to be treated independently as a simple beam. This may be accomplished mathematically by introducing hinges at the supports so that no moment can be transferred from one span to another.

Since there is a direct analogy between loads, shears, and moments in the conjugate beam to angle changes, slopes, and deflections in the real beam, it is relatively easy to find the slope and deflection at any point in the real beam.

By applying a moment M to the simple beam at point B as shown in Fig. 1b and loading the conjugate beam with the M/EI diagram, the slope at any point in the real beam may be evaluated.

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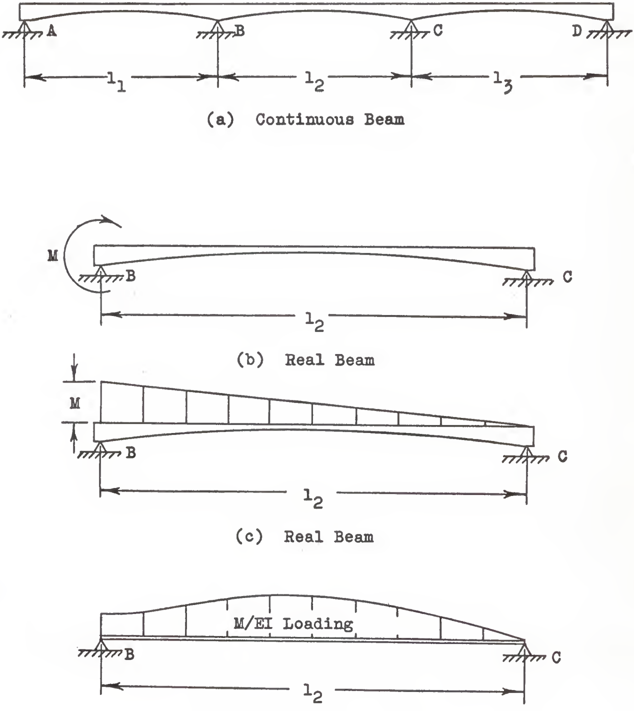
3 Ibid., p. 493.



(a) Continuous Beam

(b) Real Beam

(c) Real Beam

(d) Conjugate Beam

FIG. 1 - CONJUGATE BEAM ANALYSIS

The M/EI loading which is applied to the conjugate beam (Fig. 1d) may be divided into a series of concentrated loads in order to evaluate the non-uniform loading. Physically, the concentrated loads derived from the M/EI curve are concentrated angle changes or abrupt changes in slope of the elastic curve. Geometrically, the procedure is the same as defining the elastic curve as a series of straight lines between the concentrated loads rather than a smooth curve.[4]

The M/EI loading may be divided into any number of intervals depending on how many points of the influence line are to be evaluated. The number of intervals used in this report is ten, all of which are equal and of length λ.

Thus, the M/EI loading on span BC is divided into ten equal divisions and replaced by a series of concentrated loads, known as elastic weights. The formulas for the elastic weights are given by N.W. Newmark[5] and are shown in Fig. 2.

When using the conjugate beam method the usual sign convention is to treat loads, which are acting downward on the beam, as negative loads and when the elastic curve of the real beam is concave upward, the beam is considered to be subjected to positive moment. Therefore, when the moment curve of the

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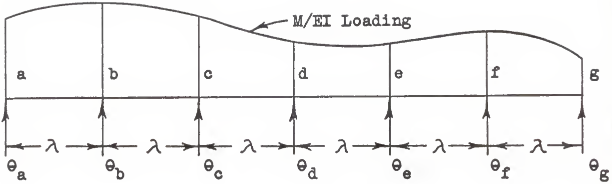
[4] "Structural Mechanics," by Samuel T. Carpenter, John Wiley & Sons, Inc., New York, N.Y., 1960, pp. 32-36.

[5] "Numerical Procedure For Computing Deflections, Moments, and Buckling Loads," by N.W. Newmark, Transactions, ASCE, Vol. 108, 1943.



$\theta_a = \lambda/24(7a+6d+c)$

$\theta_b = \lambda/12(a+10b+c)$

$\theta_c = \lambda/12(b+10c+d)$

$\theta_d = \lambda/12(c+10d+e)$

$\theta_g = \lambda/24(3e+10f+g)$

FIG. 2 - FORMULAS FOR THE ELASTIC WEIGHTS

real beam is positive, the M/EI curve should be treated as a downward load on the conjugate beam.

Consider the positive y axis as downward and the positive x axis extending from left to right. The positive shear in the conjugate beam then corresponds to positive slope in the real beam and positive bending moment agrees with positive or downward deflection.

With this sign convention and the conjugate beam loaded with the concentrated "angle changes" as shown in Fig. 3a, the slope at any point in the real beam may now be determined. By taking moments about point C of the conjugate beam the reaction R_B may be determined, thus determining θ_B.

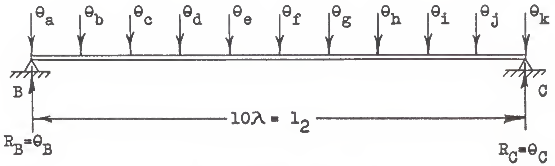
The slope at any point is found by subtracting from θ_B the "angle changes" from left to right, as shown by the slope diagram for the real beam in Fig. 3b.

By using certain relationships of end slopes and moments applied to the beam, the influence lines for the fixed-end moments and the elastic properties of the beam may be found.

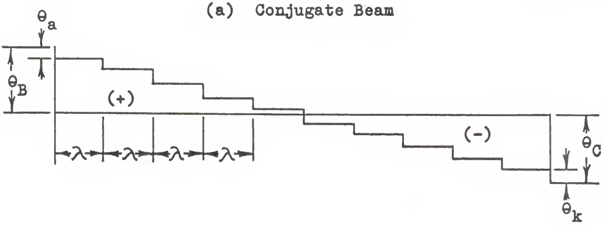
Betti's Law

With the aid of Betti's Law[6], the Müller-Breslau principle is shown to be valid for determining the influence lines. Betti's Law may be stated as follows: The external virtual work done by

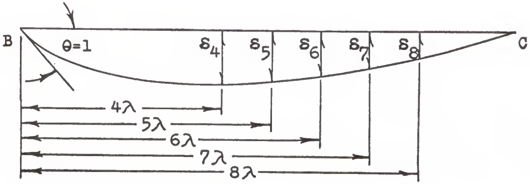
6 "Elementary Structural Analysis," by Charles Norris and John Wilbur, McGraw-Hill Book Co., New York, N.Y., 2d Ed., 1960, p. 390.



(a) Conjugate Beam



(b) Slope Curve



(c) Influence Line

FIG. 3 - INFLUENCE LINE FOR THE FIXED-END MOMENT AT B

a system of P forces during the deformation caused by a system of Q forces is equal to the external virtual work done by the Q forces during the deformation caused by the P forces. This law is valid for any structure of elastic material which has unyielding supports and is at constant temperature during the application of the forces.

The validity of Betti's Law is shown by first applying a force P, then a force Q to the system in Fig. 4a. The total work done by the system due to the P and Q forces is then equated to work done by the same forces when their order of application is reversed and the following equation results.

$$\tfrac{1}{2}P(\Delta_{np})+P(\Delta_{nq})+\tfrac{1}{2}Q(\Delta_{nq})=\tfrac{1}{2}Q(\Delta_{nq})+Q(\Delta_{np})+\tfrac{1}{2}P(\Delta_{np})$$

subtracting like terms from both sides of the equation gives:

$$P\ (\Delta_{nq}) = Q\ (\Delta_{np})$$

Noting that the first subscript in Δ_{nq} denotes the point of application of the force and the second subscript denotes the force applied, then Betti's Law has been shown to be valid.

Müller-Breslau's Principle

The Müller-Breslau principle may now be stated as follows: The ordinates of an influence line for any stress element (such as axial force, shear, moment, or reaction) of any structrue are proportional to the ordinates of the deflection curve obtained by removing the restraint corresponding to that stress element and introducing a deformation into the primary structure which remains.

Although this principle can be demonstrated for all stress elements, only the fixed-end moments for a pinned-end beam will be considered since this is the only influence line to be used in this report.

First a unit load at point m and a moment M, sufficient to prevent rotation at point B, is applied to the simple beam shown in Fig. 4b. Then a moment M' is applied at point B causing a rotation θ_{bb} at B and a deflection Δ_{mb} at point m.

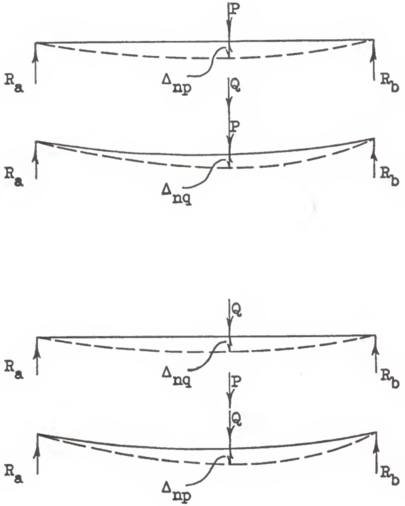
Considering the unit load, the moment M, and their reactions as the P forces on the system and the moment M' along with its reactions, shown in Fig. 4b, as the Q forces on the system, Betti's Law may be applied as follows:

$$R_a(0)+R_b(0)+1(\Delta_{mb})-M(\theta_{bb})=R_a'(0)+R_b'(0)+M'(0)$$

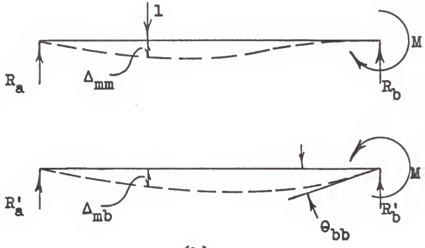
Thus;
$$1(\Delta_{mb})=M(\theta_{bb})$$
$$M = (\Delta_{mb}/\theta_{bb})\cdot 1$$

From this equation, it is apparent that the moment M is proportional to the deflection Δ_{mb} caused by the rotation θ_{bb}. Note, that if the rotation θ_{bb} were unity, then the moment M is exactly equal to the deflection Δ_{mb}.

Since the moment at any point in the conjugate beam is equal to the deflection of the corresponding point in the real beam, the ordinates to the influence line for the fixed-end moment of point B may be determined. First the ordinates of the slope diagram, shown in Fig. 3b, are divided by θ_B, thus reducing the rotation at B to unity.



(a)



(b)

FIG. 4 - DEVELOPMENT OF MULLER-BRESLAU PRINCIPLE

Influence Line Ordinates

The influence line ordinate for the moment at B due to a unit load at any tenth point is found by accumulating the area under the shear diagram as follows:

$$\delta_1 = \lambda/\theta_B(\theta_B-\theta_a)$$
$$\delta_2 = \lambda/\theta_B(\theta_B-\theta_a-\theta_b) = \delta_1 - \lambda/\theta_B(\theta_b)$$
$$\delta_3 = \lambda/\theta_B(\theta_B-\theta_a-\theta_b-\theta_c) = \delta_2 - \lambda/\theta_B(\theta_c)$$
$$\vdots$$
$$\delta_i = \delta_{i-1} - \lambda/\theta_B(\theta_{i-1})$$

δ_i being the fixed-end moment at B due to a unit load at i, where i is $i\lambda$ distance from point B. The influence line for the fixed-end moment at B, with the far end pinned, is shown in Fig. 3c.

For non-symmetrical beams the procedure is applied to both ends, thus determining the influence lines for the moment at B and the moment at C. If the beam is symmetrical about the ℄, the influence line for the moment at B and C will be the same; that is, the fixed-end moment at B due to a unit load at the first tenth point will equal the fixed-end moment at C due to a unit load at the ninth point. Also the moment at B due to a unit load at the second tenth point is equal to the moment at C due to a unit load at the eighth point.

Once the influence lines for the fixed-end moment at B with end C pinned and the fixed-end moment at C with end B pinned have been determined, it is required to find the influence line

for the fixed-end moment at B and C for both ends of the beam fixed against rotation.

In Fig. 5a, the moment H_B due to a unit load at m and with the end conditions as shown, is by definition, equal to the influence line ordinate at point m for the fixed-end moment at B. Also, the moment H_c, in Fig. 5b, is equal to the influence line ordinate for a unit load at point m.

Using the notation C_B and C_C for the moment, at B and C respectively, due to a unit load at point m and with both ends fixed against rotation it is possible to express these moments in terms of H_B, H_C, and the carry-over factors. By applying the moment C_C, to end C, in the opposite direction of the fixed-end moment the beam is then free to rotate at point C and the moment at that end becomes zero. Note, that by applying the moment C_C, the moment at end B is increased by the amount $r_{CB} \cdot C_C$, where r_{CB} is the carry-over factor from end C toward end B. The moment, required to fix end B against rotation, due to a unit load at point m is then equal to C_B plus $r_{CB} \cdot C_C$.

Since the moment H_B is the fixed-end moment at point B with end C free to rotate then the moment H_B must equal the same moment C_B plus $r_{CB} \cdot C_C$.

Thus:

$$H_B = C_B + r_{CB} \cdot C_C \tag{1}$$

By the same reasoning and application of moments to the beam in Fig. 5b, the fixed-end moment at end C, for the conditions shown in the first diagram, may also be expressed as follows:

$$H_C = C_C + r_{BC} \cdot C_B \tag{2}$$

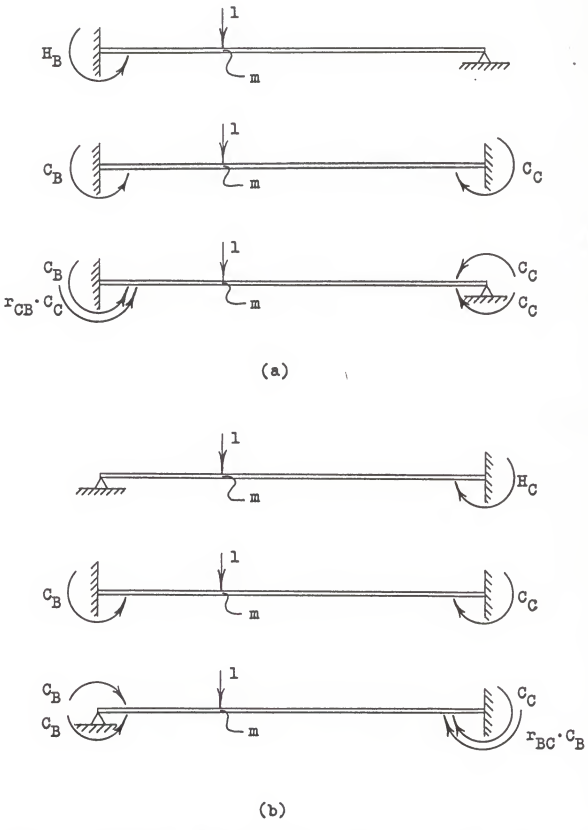


FIG. 5 - FIXED-END MOMENTS FOR A UNIT LOAD AT POINT "m"

Solving equations (1) and (2) simultaneously, the following expression for the moment at B and C will result.

$$C_B = \frac{H_B - r_{CB} \cdot H_C}{(1 - r_{BC} \cdot r_{CB})} \quad (3)$$

$$C_C = \frac{H_C - r_{BC} \cdot H_B}{(1 - r_{BC} \cdot r_{CB})} \quad (4)$$

Since the ordinate at point m of an influence line for the moment C_B is, by definition, equal to the moment at B due to a unit load at point m, then the ordinates for that influence line may be found by equation (3). Therefore, the ordinate of the influence line for the moment C_B or C_C at any tenth point in the beam is found by substituting the values for H_B and H_C, at the same tenth point, into equation (3) or (4).

Stiffness and Carry-Over Factors

For an unsymmetrical beam with a variation in the moment of inertia the stiffness and carry-over factors may be formalized in the following manner. In the upper diagram of Fig. 6a the moment S_B is applied at B while end C is fixed. Then, by definition, the rotation at end B is equal to 1 radian, and the moment induced at end C is equal to $r_{BC} \cdot S_B$. In the same manner, the moment S_C is applied to end C causing a unit rotation at that end as shown in Fig. 6b.

With the usual ij convention for subscripts, a unit moment is first applied at end B causing a rotation ϕ_{BB} at B and a

rotation ϕ_{CB} at end C. (lower diagram Fig. 6a) Also a unit moment is applied at end C, causing the rotations ϕ_{BC} and ϕ_{CC} in Fig. 6b. By superimposing or combining the rotations at end C caused by the moment S_B, applied at end B, the following equations will result.[7]

$$S_B \phi_{CB} - r_{BC} \cdot S_B \phi_{CC} = 0$$

Thus,

$$r_{BC} = \frac{\phi_{CB}}{\phi_{CC}}$$

According to Maxwell's reciprocal relation ϕ_{CB} must equal ϕ_{BC}, hence

$$r_{BC} = \frac{\phi_{BC}}{\phi_{CC}}. \tag{5}$$

Since the rotation at end B is zero, Fig. 6b, it can also be proven that

$$S_C \phi_{BC} - r_{CB} \cdot S_C \phi_{BB} = 0$$

or

$$r_{CB} = \frac{\phi_{BC}}{\phi_{BB}}.$$

Applying Maxwell's reciprocal relation again gives

$$r_{CB} = \frac{\phi_{CB}}{\phi_{BB}}. \tag{6}$$

Therefore, the carry-over factors r_{BC} and r_{CB} are found to be the ratio of the rotations produced by a unit moment at B and C.

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7 "Structural Mechanics," by Samuel T. Carpenter, John Wiley & Sons, Inc., New York, N.Y., 1960, pp. 324-326.

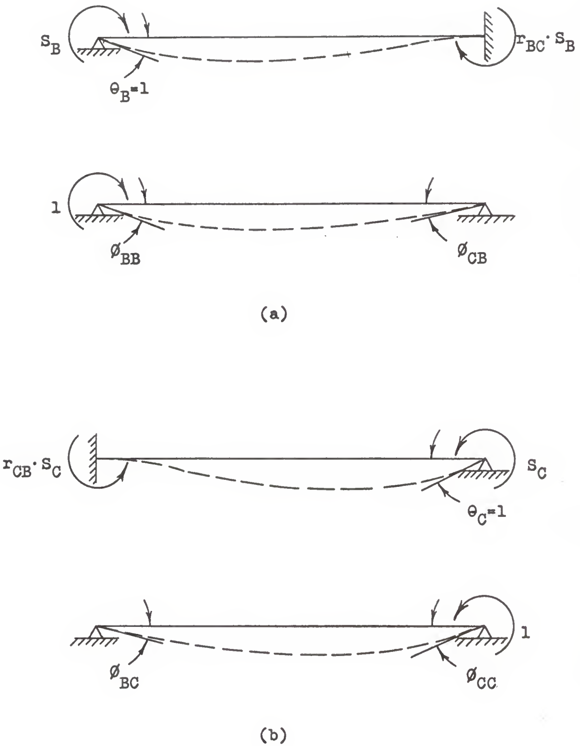

FIG. 6 - END SLOPES CAUSED BY UNIT AND STIFFNESS MOMENTS

Note, that the ratio of the end slopes for any arbitrary applied moment would provide the same result.[8]

With the moment M applied at end B, Fig. 1b, the resulting end slopes or rotations were found to be θ_B and θ_C for ends B and C respectively. Applying equation (6), the carry-over factor r_{CB} may be found as follows:

$$r_{CB} = - \frac{\theta_B}{\theta_C} \tag{7}$$

In order to evaluate the carry-over factor r_{BC}, a moment must be applied to end C of the same beam and the end slopes determined. The negative ratio of these end slopes, note equation (5), is then equal to r_{BC}.

Modified Stiffness

When a member has its far end pinned or hinged as in Fig. 1b, instead of being fixed, less moment is required to rotate the end through a given angle. The moment required to rotate the tangent at B through a unit angle (1 radian) is defined as the modified stiffness. The modified stiffness S_B^M can be expressed in terms of the stiffness S_B and the carry-over factors r_{BC} and r_{CB} as follows;

$$S_B^M = S_B(1 - r_{BC} \cdot r_{CB}) \tag{8}$$

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[8] Ibid., pp. 324-326.

Since the moment M, applied to end B in Fig. 1b, caused a rotation equal to θ_B at point B, the moment S_B^M required to rotate end B through an angle of unity is

$$S_B^M = \frac{M}{\theta_B} . \tag{9}$$

Combining equations (8) and (9) gives:

$$S_B = \frac{S_B^M}{(1 - r_{BC} \cdot r_{CB})} = \frac{M}{(1 - r_{BC} \cdot r_{CB})\, \theta_B} . \tag{10}$$

Similarly, the stiffness at end C of the beam in Fig. 1b may be found by introducing a moment at that end, causing a rotation at C. Solving for the stiffness S_C in the same manner gives:

$$S_C = \frac{M}{(1 - r_{BC} \cdot r_{CB})\, \theta_C} . \tag{11}$$

Note, that the moment M in equation (11) may be the same as the moment induced at end B, but the rotation θ_C is not the same rotation caused by the moment M when it is applied at end B.

Influence Lines Obtained by Superposition

The influence lines for the moment at any support may be obtained by superimposing the effects of the distributed moment, due to a unit moment at each end of the span and the moment due to a unit load in the span. Since a load in span BC induces fixed-end moments at B and C, it is necessary to determine the moment at B and C due to the unit moments applied as shown in Fig. 7b.

The distributed moments due to the unit fixed-end moment acting first at end B, then at end C, are used to combine the moments due to a unit load in the span. Thus,

$$M_B = +0.432\ C_B + 0.197\ C_C \tag{12}$$

$$M_C = -0.197\ C_B - 0.432\ C_C \tag{13}$$

The positive sign for the moments, or influence line ordinates, at B corresponds to the sign convention used in moment distribution. By the sign convention used in this report the moment over the support B is negative. Similarly, the negative sign for the moments at C corresponds to negative moment over the support.

EXAMPLE PROBLEM

To illustrate the procedure used to determine the influence lines for the moment at the supports, the following problem has been selected. Assume a three-span continuous beam (Fig. 7a) with parabolic haunches and span lengths of 60 feet. For simplicity, the spans are of equal length and have the same dimensions. Since each span is symmetrical about the mid-span, the stiffness, carry-over factors, and the influence lines for the fixed-end moments are the same for all spans.

Based on a depth of 4 ft. over the supports, 2 ft. at the mid-span, and a width of 1 ft., the moment of inertia is calculated for each of the tenth points in the span B-C. The values appear in the second column of Table I. Next a moment of ten units is applied at point B as shown in Fig. 1b. Since

the beam is homogeneous, the value for the modulus of elasticity (E) is a constant; therefore it is replaced by unity in the calculations.

The M/EI values are calculated for each tenth point in order to determine the values for the elastic weights. (Columns 3 and 4 in Table I) With these values, the slope at each tenth point is recorded in the fifth column.

Substituting the values for r_{BC}, r_{CB}, H_B, and H_C into equation (3) results in the ordinates of the influence line for the moment C_B. Since the member is symmetrical about mid-span, the influence line for the moment C_C is the same as that for C_B.

With the influence line ordinates for all the fixed-end moments and the unit distributed moments in Fig. 7b, the influence line ordinates for the moment at B are calculated and tabulated in Table II. The general shape of this influence line is shown in Fig. 7c.

The carry-over factors, stiffness, and some of the fixed-end moments, when compared to those in Table 15 of the "Handbook of Frame Constants,"[9] are found to be quite accurate. Usually three to four significant figures are sufficient for the accuracy required in most design problems.

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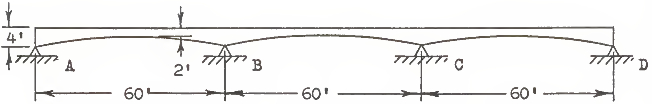
[9] "Handbook of Frame Constants," Portland Cement Association, Chicago, Ill., 1958, p. 9.

TABLE I — Influence Line Ordinates For The Fixed-End Moments

POINT	I(Ft.^{4})	M/I	ELASTIC WEIGHTS	SLOPE	H_B	H_C	C_B
				146.746			
B	5.333	1.875	9.064		0.000	0.000	0.000
				137.682			
0.1	2.941	3.060	18.624		5.631	4.094	5.386
				119.058			
0.2	1.677	4.770	28.472		10.501	8.094	7.498
				90.586			
0.3	1.041	6.724	40.005		14.206	11.784	11.637
				50.581			
0.4	0.750	8.000	47.112		16.274	14.752	11.653
				3.469			
0.5	0.667	7.500	44.166		16.416	16.416	9.697
				-40.697			
0.6	0.750	5.333	31.856		14.752	16.274	6.676
				-72.553			
0.7	1.041	2.882	17.673		11.784	14.206	3.716
				-90.226			
0.8	1.677	1.193	7.576		8.094	10.501	1.556
				-97.802			
0.9	2.941	0.340	2.296		4.094	5.631	0.359
				-100.098			
C	5.333	0.000	1.745		0.000	0.000	0.000
				-101.843			

TABLE II — Influence Line Ordinates For The Moment At B

POINT	ORDINATE	POINT	ORDINATE	POINT	ORDINATE
A	0.000	B	0.000	C	0.000
0.1	-2.325	0.1	-2.397	0.1	+1.109
0.2	-4.597	0.2	-3.546	0.2	+2.069
0.3	-6.693	0.3	-5.759	0.3	+2.799
0.4	-8.379	0.4	-6.349	0.4	+3.206
0.5	-9.324	0.5	-6.099	0.5	+3.239
0.6	-9.244	0.6	-5.180	0.6	+2.906
0.7	-8.069	0.7	-3.898	0.7	+2.321
0.8	-5.965	0.8	-2.149	0.8	+1.595
0.9	-3.198	0.9	-1.216	0.9	+0.807
B	0.000	C	0.000	D	0.000



(a) Continuous Beam

	0.342	0.658	0.658	0.342	
0.0	-1.000	0.0	0.0	0.0	0.0
	+0.432	+0.831	+0.577		
		-0.263	-0.380	-0.197	
0.0	-0.568	+0.568	+0.197	-0.197	0.0

	0.342	0.658	0.658	0.342	
0.0	0.0	+1.000	0.0	0.0	0.0
	-0.432	-0.831	-0.577		
		+0.263	+0.380	+0.197	
0.0	-0.432	+0.432	-0.197	+0.197	0.0

	0.342	0.658	0.658	0.342	
0.0	0.0	0.0	-1.000	0.0	0.0
		+0.577	+0.831	+0.432	
	-0.197	-0.380	-0.263		
0.0	-0.197	+0.197	-0.432	+0.432	0.0

(b) Moment Distribution

(c) Influence Line For The Moment At "B"

FIG. 7 - EXAMPLE PROBLEM

CONCLUSION

The numerical procedure described herein permits a simple and accurate calculation of the influence lines for the moment at the supports. This method can also be extended to continuous beams which have spans that are non-symmetrical, unequal in length, or both. The procedure may also be applied to other problems of the same mathematical nature. As previously stated, other influence lines may be determined by using the equations of statics.

In addition to determining the elastic properties of a beam and the influence lines for the moments, this procedure can be used to determine the deflection caused by some type of loading on the beam. Once the influence lines (for the moment at the supports) have been determined, the moment at any point in the beam can readily be found. Using this moment, the M/EI loading can now be applied to the conjugate beam. The deflection at any point in the real beam can be determined by the same general procedure used to determine the ordinates for the influence lines.

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DETERMINATION OF INFLUENCE LINES
AND ELASTIC PROPERTIES

by

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Influence lines for statically indeterminate structures may be obtained by several different numerical procedures, all of which are based on the principles of elementary structural mechanics. Using some of the same basic principles in a more efficient and organized procedure can reduce a somewhat complex structural problem to one which is accurately and efficiently solved.

The procedure set forth in this report is based on the Müller-Breslau Principle combined with the Moment Distribution Method and is very adaptable to beams which are continuous over more than two simple supports. Although the procedure is best suited for members with non-uniform cross-section, it is also valid for members with uniform cross-section.

When the sectional variation becomes difficult to express as a function of x, as is the case in many indeterminate beams with a variable moment of inertia, the use of elastic weights applied to the conjugate beam lends itself readily.

The modified stiffness, true stiffness, and the carry-over factors are determined for each end of each span. With these values, a unit moment applied at each end of each span can then be distributed by the Moment Distribution Method. Using the resulting moments to combine the fixed-end moment influence lines, the influence lines for the moment at any interior support may be determined. With such influence lines

determined it is relatively easy to calculate any other influence lines, for moment, shear, or reactions, by using the equations of statics.

In order to illustrate the method used in determining the influence lines for the support moments and the elastic properties an example problem is worked in detail. In the example problem, for simplicity, the span lengths are equal and each span is symmetrical about mid-span.